I Semester B.A./B.Sc. Examination, November/December 2015
(Semester Scheme) (2011-12 and Onwards) (N.S)
MATHEMATICS – I

Time : 3 Hours   Max. Marks : 100

Instruction : *Answer **all** questions.*

I. Answer **any fifteen** questions :   **(15×2=30)**

1) Find the rank of the matrix $A = \begin{pmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & 8 \end{pmatrix}$.

2) Find the Eigen values of the matrix $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$.

3) If the rank of the matrix $A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & -3 & -1 \\ a & 0 & 1 \end{pmatrix}$ is 2, find a.

4) Prove that the characteristic root of a singular matrix is zero.

5) Find the n^{th} derivative of $e^{2x} \cos 2x$.

6) Find the n^{th} derivative of $\cos^2 x$.

7) If $u = e^{2x} \sin 5y$, find u_{xy}.

8) If $u = x^3 - 4x^2y - 2xy^2 + y^3$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 3u$.

9) If $x^2 - y^2 = a^2$ find $\dfrac{dy}{dx}$ using partial differentiation.

10) If $u = 3x + 5y$, $v = 4x - 3y$, find $\dfrac{\partial(u, v)}{\partial(x, y)}$.

11) Evaluate : $\int_0^{\pi/2} \cos^6 x \, dx$.

12) If $I_n = \int_0^{\infty} x^n e^{-x} \, dx$, prove that $I_n = nI_{n-1}$.

P.T.O.

13) If α, β, γ are the angles made by a line with coordinate axes, prove that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.

14) Find the equation of the plane passing through the point (2, 4, 3) and parallel to the plane $5x - 6y + 7z = 3$.

15) Find the angle between the planes $3x - 6y + 2z + 5 = 0$ and $4x + 2y + 3z + 3 = 0$.

16) Find the equation of the line passing through the points (2, 3, 7) and (4, –5, 8).

17) Find the distance between the parallel planes $2x - 2y + z + 6 = 0$ and $2x - 2y + z + 7 = 0$.

18) Find the equation of the sphere with centre (3, –1, 2) and radius 5 units.

19) Find the equation of the right circular cone with vertex at (0, 0, 0), semi-vertical angle 30° and axis along y-axis.

20) Find k if the spheres :

$x^2 + y^2 + z^2 + 6y + 2z + k = 0$ and $x^2 + y^2 + z^2 + 6x + 8y + 4z + 20 = 0$ cut orthogonally.

II. Answer **any two** questions : (2×5=10)

1) Find the rank of the matrix A by reducing it to normal form where

$$A = \begin{pmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}.$$

2) Test the following system of equations for consistency and solve if consistent,

$x + 2y + 2z = 1$, $2x + y + z = 2$, $3x + 2y + 2z = 3$, $y + z = 0$.

3) Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ hence find A^{-1} using the theorem.

4) Find the Eigen values and the corresponding Eigen vectors of the matrix

$$A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}.$$

(4×5=20)

III. Answer **any four** questions :

1) Find the n^{th} derivative of $\frac{1}{6x^2 - 5x + 1}$.

2) If $y = \cos(m \sin^{-1} x)$, prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - m^2)y_n = 0$.

3) If $z = \sin(ax + y) + \cos(ax - y)$, prove that $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$.

4) State and prove Euler's theorem for homogeneous function of degree n in x and y.

5) If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ prove that $6\frac{\partial u}{\partial x} + 4\frac{\partial u}{\partial y} + 3\frac{\partial u}{\partial z} = 0$.

6) If $x = r\cos\theta\cos\phi$, $y = r\cos\theta\sin\phi$, $z = r\sin\theta$, prove that $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = -r^2\cos\theta$.

(2×5=10)

IV. Answer **any two** questions :

1) Evaluate : $\int_0^{\pi} x \sin^6 x \, dx$.

2) Obtain the reduction formula for $\int \cos^n x \, dx$, n is a positive integer.

3) Evaluate $\int_0^{\infty} \frac{e^{-ax} \sin x}{x} dx$, $a > 0$, by using Leibnitz's rule of differentiation under integral sign.

(4×5=20)

V. Answer **any four** questions :

1) Find the direction cosines of the two lines satisfying $l + m + n = 0$ and $2lm + 2ln - mn = 0$.

2) Derive the equation of the plane in the form $\vec{r}.\hat{n} = p$.

3) Find the equation of the plane passing through the points (–4, 4, 4), (4, 5, 1) and (0 –1, –1).

4) Find the equation of the plane passing through the points (1, 1, 1) and perpendicular to the planes $x - 3y + 5z + 4 = 0$ and $3x - y + 7z + 7 = 0$.

5) Show that the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{1}$ and $\frac{x-2}{3} = \frac{y-2}{2} = \frac{z-6}{4}$ are coplanar. Find the equation of the plane containing them.

6) Find the shortest distance between the lines $\frac{x+1}{2} = \frac{y-2}{3} = \frac{z-2}{-2}$ and $\frac{x-2}{2} = \frac{y-8}{2} = \frac{z+1}{-1}$.

VI. Answer **any two** questions : (2×5=10)

1) Find the equation of the sphere passing through the points (3, 0, 0), (0, –1, 0) and (0, 0, –2) and having its centre on the plane $2x + 2y + 4z - 1 = 0$.

2) Find the equation of the right circular cone with vertex at (3, 1, 2) semi-vertical angle $\cos^{-1}\left(\frac{1}{\sqrt{7}}\right)$ and axis has direction ratios 1, 2, 3.

3) Find the equation of the right circular cylinder of radius 2 units and whose axis is $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-3}{5}$.